Investigation of possible microcavity effect on lasing threshold of nonradiative-scattering-dominated semiconductor lasers

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The effect of enhanced rate of spontaneous emission on gain and lasing threshold of semiconductor microcavity lasers has not been discussed clearly. Some reports have suggested that the lasing threshold in microcavities could possibly be lowered due to the so-called *Purcell effect*. Here, we argue that gain in weakly coupled semiconductor cavities is a local phenomenon, which occurs due to stimulated emission induced by an electromagnetic excitation and remains unaffected by the cavity boundary conditions. Hence, the Purcell effect in microcavities filled uniformly with a gain medium should not lead to a reduction in the laser's threshold pump density, provided radiative scattering is not the dominant relaxation mechanism in the excited state. A systematic experimental investigation of laser threshold in parallel-plate semiconductor microcavity terahertz quantum-cascade lasers of different dimensions was found to be in accordance with our arguments. © *2012 American Institute of Physics*. [doi:10.1063/1.3678595]

The fundamental processes of spontaneous and stimulated emission of radiation are known since 1917 when Einstein laid the foundation for a quantum theory of radiation.[1] It was suggested early that the spontaneous emission rate could be altered (enhanced or inhibited) in a resonant optical cavity,[2,3] which was verified experimentally.[4] Spontaneous emission is a manifest of the interaction of matter with vacuum-field fluctuations. The rate of spontaneous emission depends on the density of electromagnetic modes in the cavity and the intensity of the vacuum-field, which makes it dependent on the dimensions and quality-factor of the cavity. Hence, the rate of spontaneous emission can be enhanced in an optical microcavity[5] that has at least one of its dimensions smaller than the wavelength. On the other hand, the rate of stimulated emission per unit energy spectral density is an inherent characteristic of the material, irrespective of the cavity's boundary conditions. The overall optical gain is proportional to the inverted population density as well as the rate of stimulated emission at a particular frequency of radiation. Hence, as long as the inverted population density is predominantly determined by nonradiative processes, that is, the nonradiative lifetime τ_{nr} is shorter than the spontaneous emission lifetime τ_{sp}, the optical gain should remain unaffected by the enhancement in rate of spontaneous emission due to the Purcell effect. The condition $\tau_{nr} < \tau_{sp}$ applies to all intersubband (quantum-cascade) lasers[6] even at cryogenic temperatures, which are the subject of the experimental investigation in this letter. This condition may also be applicable to some types of interband lasers such as the type-II mid-IR lasers[7] in which Auger recombination could be significant or room-temperature photonic-crystal nanocavity lasers based on quantum-dots[8] or quantum-wells[9] in which the rate of surface recombination can be the dominant relaxation mechanism below threshold.

In the original paper by Einstein,[1] it was shown that the B coefficient (i.e., the rate of stimulated emission per unit energy spectral density) is proportional to the A coefficient (i.e., the rate of spontaneous emission into all available optical modes), which might lead to the conclusion that an enhanced rate of spontaneous emission by a microcavity should result in an enhanced rate of stimulated emission and thus a higher optical gain for a given population inversion. In this case, the required population-inversion density at the threshold of a laser may be reduced by the microcavity effect that enhances A. Such a reduction will certainly have important technological impacts. However, here we argue that the relationship between A and B is unique for a given system that includes the oscillator and its surrounding cavity. It turns out that while B is only dependent on the oscillator medium, A however also depends on the surrounding cavity. As a result, alteration of A by using microcavities will have no impact on B, and consequently on gain and lasing threshold.

To begin with a simple discussion, treating lasers as classical oscillators, the lasing threshold (oscillation condition) is defined as the round-trip power gain being unity for the lasing mode, that is,

$$g_{mod} = \Gamma_{mod} \quad g_{mat} = \alpha_{mod},$$

or equivalently,

$$g_{mat} = \alpha_{mod}/\Gamma_{mod}. \tag{1}$$

In Eq. (1), g_{mod} is the modal gain (typically specified in per unit length), Γ_{mod} is the dimensionless mode-confinement factor, g_{mat} is the material gain, and α_{mod} is the modal loss. In the second expression of Eq. (1), only the right-hand side depends on the cavity, while the material gain on the left-hand side is simply proportional to the imaginary part of the linear dielectric susceptibility $\chi = \chi' + i\chi''$ of the gain medium ($g_{mat} \approx \chi''\omega/(n_r c)$, where n_r is the refractive index of

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the medium). Thus, the material gain $g_{\rm mat}$, which is proportional to the pumping rate density, is purely a local property and completely independent of the boundary conditions of the cavity. In essence, the rate of spontaneous emission can certainly be altered by changing the electromagnetic mode densities using a microcavity. However, the gain (loss) reflects the amplification (attenuation) of a single electromagnetic mode (which can be seen clearly from the definition of gain g and loss α as $\frac{dI(z)}{dz} = (g-\alpha)*I(z)$ for light intensity I propagating along z), regardless whether it is enhanced or suppressed by a cavity, hence the independence of the gain (loss) on cavity parameters. For the right-hand side of Eq. (1), a microcavity with a reduced cavity volume (but with the same boundary conditions) will usually increase the modal loss $\alpha_{\rm mod}$ and reduce the mode-confinement factor $\Gamma_{\rm mod}$. An exception to this is when the active region does not completely fill the cavity, in which case reducing the cavity dimensions may increase $\Gamma_{\rm mod}$; however, such lasers are excluded from this discussion. Based on this straightforward discussion, a microcavity will have no positive effect on the reduction of the pump threshold density of a laser oscillator, provided that the relaxation of the carriers in the excited state is not dominated by the rate of spontaneous emission. It should be pointed out, however, that for oscillators based on nonlinear optical processes such as optical parametric oscillators and Raman lasers, the material gain depends on the strength of the local radiation field of the pump, which can certainly be enhanced by the cavity boundary conditions. Thus, it is possible that the pump threshold density can be reduced for those oscillators by the microcavity effect.

The results presented in this paper apply to semiconductor lasers with poor radiative efficiencies, i.e., where nonradiative scattering is the dominant relaxation mechanism in the upper radiative state. Furthermore, we exclude lasers in the strong-coupling regime in which the vacuum-Rabi energy splitting is larger than the cavity resonance linewidth.[10] Also, our conclusions do not apply to lasers in which the cavity resonance linewidth is broader than that of the material (gain-medium) resonance (i.e., "bad-cavity" lasers[11]).

It is important to distinguish a possible Purcell effect on the lasing threshold reduction from that of purely classical effect, since the Purcell factor, which is $\propto Q/V$ where Q is the quality-factor of the microcavity and V is the modal volume, does have impact on the threshold pump. As has been widely used in literature, the pump has been defined as the total pump power or injection current. Using such metrics, the threshold pump will certainly decrease with the volume of the cavity. Furthermore, since $Q \propto 1/\alpha_{\rm mod}$, the pump threshold will also decrease with $1/Q$. However, both of these threshold reduction mechanisms are purely classical effects, even though they may superficially appear as $P_{\rm th} \propto V/Q$, but they really have nothing to do with the QED Purcell effect. In order to separate Purcell effect from those classical effects, we will focus on the measure of pump density (in [W/m^3]) at the threshold to eliminate the classical volume effect. Furthermore, in order to avoid the ambiguity associated with Q, we have chosen a microcavity system that has demonstrated a significant Purcell enhancement of spontaneous emission rate, while totally independent of Q.

We now describe our experiments with terahertz quantum-cascade lasers (QCLs)[12] formed in microcavities, with metal-metal parallel-plate waveguides for mode confinement[13] which is shown schematically in Fig. 1(a). Large Purcell enhancement (by a factor of ~50) of the rate of spontaneous emission was recently reported in similar terahertz microcavities with the quantum-cascade gain medium,[14] which was our primary motivation for choosing such type of lasers for the experiment.

At terahertz frequencies, a parallel-plate cavity of the type shown in Fig. 1(a) becomes a microcavity in the vertical dimension for the typical thicknesses ($d \sim 10\,\mu$m). Consequently, the rate of spontaneous emission due to an intersubband radiative transition in such a microcavity is modified. The derivation for the A and B coefficients for intersubband transitions is standard and follows from the Fermi golden rule[15] to yield

$$B = \frac{\pi e^2 z_{ul}^2}{\epsilon_{\rm r}\epsilon_0 \hbar^2} = \frac{\pi e^2}{2m^* \epsilon_{\rm r}\epsilon_0} \frac{f_{ul}}{\hbar\omega_{ul}}, \tag{2}$$

and

$$A = B\ \hbar\omega_{ul}\ D_{\rm cav}(\omega_{ul}), \tag{3}$$

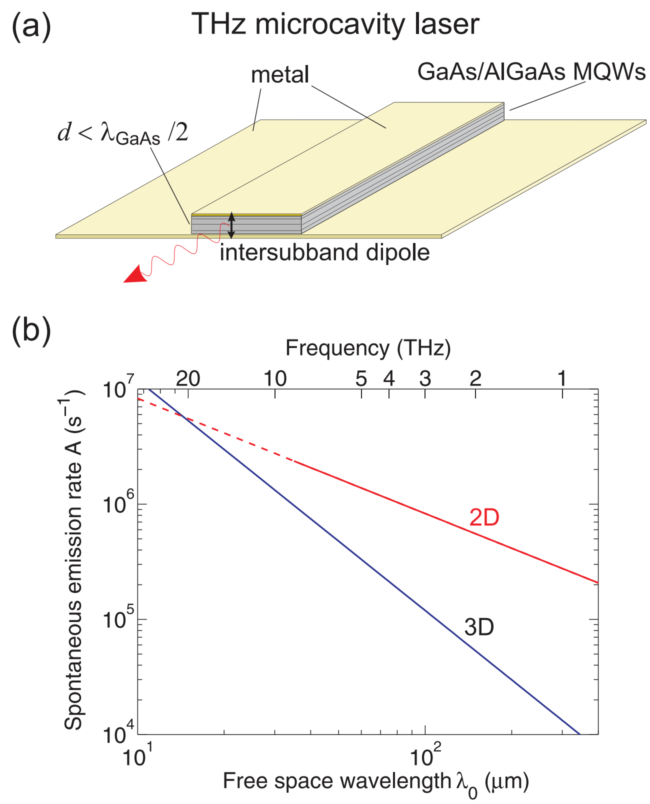

FIG. 1. (Color online) (a) Schematic showing a parallel-plate metal-metal cavity enclosing a quantum-cascade laser gain medium. At terahertz frequencies, for a cavity thickness $d < \lambda_0/(2n_{\rm r})$, the density of optical modes is enhanced from that in the bulk. (b) Spontaneous emission rate A due to an intersubband transition in GaAs/AlGaAs quantum-wells corresponding to an oscillator strength of unity and a cavity thickness of $d = 3\,\mu$m. Calculated values for the bulk material (3D) and that in the parallel-late microcavity (2D) are plotted. The dashed line indicates wavelengths for which the 2D calculation is no longer valid since $d < \lambda_0/n_{\rm r}/2$ is not satisfied and the parallel-plate cavity is no longer a pure 2D microcavity.

where e is the unit electronic charge, ϵ_r is the relative permittivity of the medium, ω_{ul} is the frequency corresponding to the radiative transition from subband $u \rightarrow l$, z_{ul} is the dipole-matrix element for the radiative transition in the units of length, $f_{ul} \equiv 2m^* \omega_{ul} z_{ul}^2/\hbar$ is the dimensionless oscillator strength (where m^* is the effective mass of the carriers), and $D_{\text{cav}}(\omega)$ is the density of optical modes in the cavity (per unit volume per unit frequency). Note that the B coefficient is independent of any cavity parameters, whereas the A coefficient is linearly related to the density of optical modes at frequency ω_{ul}, which can be enhanced in a microcavity. At long wavelengths, the radiation wavevector in the vertical dimension is negligible and the density of modes is determined from a 2D calculation as[16]

$$D_{\text{cav}}^{\text{2D}} = \frac{3}{4}\frac{(\lambda_0/n_r)}{d} D_{\text{cav}}^{\text{3D}}, \tag{4}$$

where $D_{\text{cav}}^{\text{3D}}(\omega) = \omega^2 n_r^3/(3\pi^2 c^3)$, which includes a factor of 1/3 due to the intersubband polarization selection rule that requires the electric-field to be polarized perpendicular to the quantum-wells to cause a radiative transition. The enhanced rate of spontaneous emission A for a 2D microcavity is plotted in Fig. 1(b) for an example case of an oscillator strength f_{ul} of unity for the GaAs/AlGaAs material system ($\epsilon_r \approx 13$, $m^* \approx 0.067 m_0$).

Since the rate of spontaneous emission varies inversely with the thickness of the cavity d as per Eq. (4), we have performed systematic experiments to investigate the effect of varying the thickness of the cavity on the laser's threshold current density J_{th}, which is directly related to the pump power density $P_{\text{th}} = J_{\text{th}} \cdot E$, where E is the electrical bias field in operating conditions. The active gain medium and the waveguide fabrication technique utilized for these experiments is similar to that reported in Ref. 17. Fabry-Pérot ridge lasers of three different thicknesses (10, 5.1, and 2.8 μm) were processed from a wafer grown by molecular-beam epitaxy, designed for operation at 2.9 THz (design FL178C-M10, wafer EA1252)[15] based on a four-well resonant-phonon depopulation design scheme. Different cavity thicknesses were attained by wet chemical etching of the active region that had an original as-grown wafer thickness of 10 μm. According to Eq. (4), a Purcell enhancement by a factor of $\sim$3.6 is expected from the thinnest cavity ($d = 2.8\ \mu$m) as compared to the thickest one ($d = 10\ \mu$m).

The experimental results in pulsed operation from the 2.9 THz QCLs with three different cavity thicknesses are shown in Fig. 2. Since the heavily doped ohmic contact layer is etched away in devices with thicknesses of 5.1 and 2.8 μm, there is a large voltage drop at the Schottky contact so the total voltage does not scale linearly with the thickness. As a result, the most meaningful measure of the pumping level is the current density J [A/cm^2], from which the pumping power density $J \cdot E$ can be inferred and the electric field E should be the same for all the three devices at a given band alignment. As can be seen from the J-V curves in Fig. 2, the maximum current density J_{max} at the onset of negative differential resistance (NDR) is approximately the same for all the three devices (within the normal fluctuations from device to device). The unchanged J_{max} in all the three devices indi-

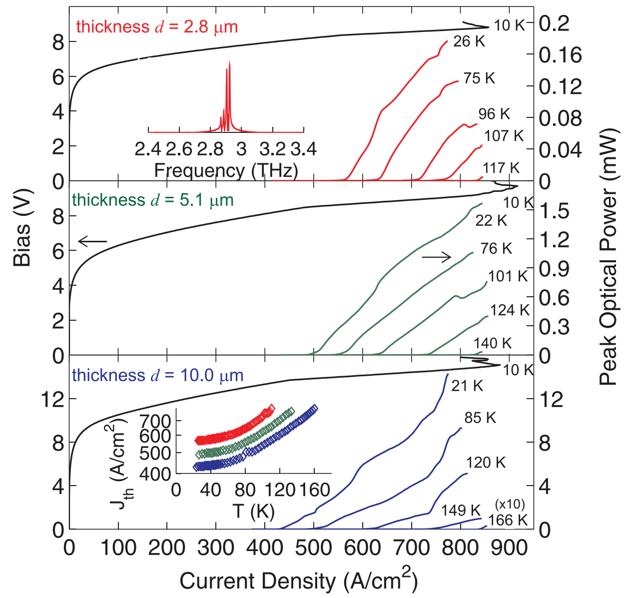

FIG. 2. (Color online) Temperature behavior of quantum-cascade lasers operating at 2.9 THz as a function of different cavity thicknesses. Pulsed light-current (L-I) characteristics from a 2.8 μm × 75 μm × 1.03 mm (upper panel), a 5.1 μm × 55 μm × 1.03 mm (middle panel), and a 10.0 μm × 95 μm × 1.05 mm (lower panel) ridge laser, respectively, measured with a Ge:Ga photodetector, and plotted for different heat-sink temperatures. Peak optical power was detected by a calibrated thermopile detector. The upper inset shows a typical spectrum. The bottom inset shows the variation of the threshold current-density J_{th} as a function of temperature T for the three different devices (the higher threshold current-density corresponds to the laser with thinner cavity).

cates that the fabrication of thinner-cavity devices did not alter any of their physical or material properties, which could otherwise impact their electrical transport characteristics. As can be seen in Fig. 2, the threshold current-density (J_{th}) at any given temperature increases monotonically for a laser with thinner cavity, likely due to a slight increase of the cavity loss. Consequently, the maximum operating temperature T_{max}, which is the most important operation parameter for THz QCLs, decreases as the dynamic range J_{max}-J_{th} reduces. This is clearly a result in the negative, should the rate of spontaneous emission have played a role in determining the gain and therefore the lasing threshold of microcavity lasers. It may be noted that increased threshold current densities for terahertz QCLs with thinner cavities have also been reported elsewhere,[18,19] which further validate the experimental results reported here. A recently developed inductor-capacitor circuit-based microcavity laser has an extremely small mode volume and an associated large Purcell factor of $\sim$17.[20] Despite this large enhancement of spontaneous emission rate, however, the device requires the application of a strong magnetic field to reach the lasing threshold. In comparison, conventional "macro" Fabry-Perot cavity lasers fabricated from the same gain medium did not require a magnetic field to reach threshold.[21] This is further evidence that the microcavity effect has no positive impact on lowering the lasing threshold, if it is meaningfully defined as the pump power density as is done in this paper.

In conclusion, we argue that the microcavity Purcell effect, which leads to alteration of the rate of spontaneous emission, should not affect the gain and the lasing threshold of semiconductor lasers with poor radiative efficiencies, since gain is a local property of the material whereas spontaneous emission also depends on the cavity's boundary conditions. Systematic experiments were performed on terahertz quantum-cascade lasers with parallel-plate metal-metal microcavities for different cavity dimensions, and no reduction in threshold-densities was observed despite an expected significant Purcell enhancement of the rate of spontaneous emission in such cavities.

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